ICS 271
Fall 2018
Instructor : Kalev Kask
Homework Assignment 5
Due Sunday November 11

1. (15) Consider a vocabulary with four propositional variables, A, B, C and D. How many models (satisfying true/false assignments) are there for the following sentences:

   (a) $B \wedge A$

   (b) $\neg A \vee \neg B \vee \neg C \vee \neg D$

   (c) $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$

2. (15) Consider the statement "The car is either at John's house or at Fred's house. If the car is not at Fred's then it must be at John's house."

   (a) Describe a set of propositional letters which can be used to represent these statements.

   (b) Describe the statements using a propositional formula on the propositions you described for (a).

   (c) Can you determine where is the car?

3. (10) How would you use the truth table to prove that *unit resolution* is sound.

4. (10) Convert the following propositional calculus wff into CNF form:

$$\neg[((P \vee \neg Q) \rightarrow R) \rightarrow (Q \wedge P)]$$

5. (20) Show how the Australian map coloring problem (from textbook/lectures) can be represented as a PSAT (propositional satisfiability) problem. (Hint: Introduce a propositional symbol for each territory and color; e.g. if NT_{red} is $True$, then NT is colored red, and if NT_{red} is $False$, then NT is not red. Now state the constraints of the problem in terms of these popositional symbols.)

6. (20) Use truth tables to show that the following sentences are valid, and thus that the equivalences hold. Some of these equivalence rules have standard names, which are given in the right column.

$P \wedge (Q \wedge R)$	$\Leftrightarrow$	$(P \wedge Q) \wedge R$	Associativity of conjunction
$P \wedge (Q \vee R)$	$\Leftrightarrow$	$(P \wedge Q) \vee (P \wedge R)$	Distributivity of conjunction
$\neg(P \wedge Q)$	$\Leftrightarrow$	$\neg P \vee \neg Q$	de Morgan's Law
$P \Leftrightarrow Q$	$\Leftrightarrow$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$	

7. (30) Look at the following sentences and decide for each if it is valid, unsatisfiable, or neither. Verify your decisions using truth tables, or by using the equivalences.

   (a) $Smoke \Rightarrow Smoke$

   (b) $Smoke \Rightarrow Fire$

   (c) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

   (d) $Smoke \vee Fire \vee \neg Fire$

   (e) $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

   (f) $Big \vee Dumb \vee (Dumb \Rightarrow Big)$

8. (30) Trace the behavior of DPLL on the knowledge-base in Figure 7.16 (Russell and Norvig textbook) when trying to prove Q, and compare this behavior with that of forward chaining algorithm.